A FREQUENCY-DEPENDENT FDTD METHOD FOR INDUCED-CURRENT CALCULATIONS FOR A HETEROGENEOUS MODEL OF THE HUMAN BODY

O. P. Gandhi, J. Y. Chen, C. M. Furse

Department of Electrical Engineering
University of Utah
Salt Lake City, Utah 84112

ABSTRACT

A weakness of the FDTD method is that dispersion of the dielectric properties of the scattering/absorbing body is often ignored and frequency-independent properties are generally taken. While this is not a disadvantage for CW or narrow-band irradiation, the results thus obtained may be highly erroneous for short pulses where ultrawide bandwidths are involved. We have developed a new differential equation approach which can be used for general dispersive media for which $\varepsilon^*(\omega)$ and $\mu^*(\omega)$ may be expressible in terms of rational functions, or for human tissues where multiterm Debye relaxation equations must generally be used. The method is illustrated by means of one- and three-dimensional examples of media for which $\varepsilon^*(\omega)$ is given by a multiterm Debye equation and for a dispersive model of the human body.

INTRODUCTION

The finite-difference time-domain (FDTD) method has previously been used for calculations of SARs and induced currents for whole-body or partial-body exposures due to spatially uniform or nonuniform (far-field or near-field) sinusoidally varying EM fields or for transient fields such as those for an electromagnetic pulse [1-3]. A weakness of the previously used FDTD algorithm is that the dispersion of the tissue's dielectric properties is ignored and frequency-independent properties are taken. While this is permissible for continuous-wave or narrow-band irradiation, the results may be highly erroneous for short pulses where ultrawide bandwidths are involved. In some recent publications, procedures are described for one- and two-dimensional problems for media for which the complex permittivity $\varepsilon^*(\omega)$ may be described by a single-order Debye relaxation equation or a modified version thereof [4, 5]. These procedures, based on a convolution integral describing $\mathbf{D}(t)$ in terms of $\mathbf{E}(t)$, cannot be extended to human tissues where multiterm Debye relaxation equations must generally be used

In this paper we describe a differential equation approach which should be easy to implement for general dispersive media for which permittivity and/or permeability can be described in the frequency domain by means of rational functions. We illustrate the use of this approach by one-dimensional and three-dimensional examples of media for which the complex permittivities are given by a multiterm Debye equation and for an approximate two-thirds muscle-equivalent model of the human body.

THE DIFFERENTIAL-EQUATION-BASED (FD)^{2}TD METHOD

The time-dependent Maxwell's curl equations used for the FDTD method are:

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} = -\mu \frac{\partial \mathbf{H}}{\partial t} \tag{1}$$

$$\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} \tag{2}$$

where the displacement vector $\mathbf{D}$ is related to the electric field $\mathbf{E}$ through the complex permittivity $\varepsilon^*(\omega)$ of the local tissue by the following equation:

$$\mathbf{D} = \varepsilon^*(\omega)\ \mathbf{E} \tag{3}$$

Since Eqs. 1 and 2 are to be solved iteratively in the time-domain, it is necessary to also express Eq. 3 in the time-domain. In the procedure proposed recently in the literature [4, 5], one uses a convolution integral to obtain $\mathbf{D}(t)$ in terms of $\mathbf{E}(t)$. This procedure is not, however, generalizable to media for which $\varepsilon(\tau)$ in the time-domain cannot be written in terms of exponential functions.

To circumvent this problem, we have developed a differential equation approach which should be easy to implement for general dispersive media for which permittivity and/or permeability can be described in the frequency-domain by means of rational functions. Because of our interest in bioelectromagnetic problems, we illustrate this procedure by taking an example where $\varepsilon^*(\omega)$ can be described by a two-relaxation Debye equation:

$$\varepsilon^*(\omega) = \varepsilon_0 \left[\varepsilon_\infty + \frac{\varepsilon_{s1} - \varepsilon_\infty}{1 + j\,\omega\,\tau_1} + \frac{\varepsilon_{s2} - \varepsilon_\infty}{1 + j\,\omega\,\tau_2} \right] \tag{4}$$

Substituting Eq. 4 into Eq. 3, we can write Eq. 3 in the time domain:

$$\tau_1 \tau_2 \frac{\partial^2 \mathbf{D}}{\partial t^2} + (\tau_1 + \tau_2) \frac{\partial \mathbf{D}}{\partial t} + \mathbf{D} =$$

$$\varepsilon_0 \left[\varepsilon_s\, \mathbf{E}(t) + \left[\varepsilon_{s1} \tau_2 + \varepsilon_{s2} \tau_1 \right] \frac{\partial \mathbf{E}}{\partial t} + \varepsilon_\infty \tau_1 \tau_2 \frac{\partial^2 \mathbf{E}}{\partial t^2} \right] \tag{5}$$

where ε_s is the zero (static) frequency dielectric constant given by

$$\varepsilon_s = \varepsilon_{s1} + \varepsilon_{s2} - \varepsilon_\infty. \tag{6}$$

For the (FD)^{2}TD method, we solve Eqs. 1 and 2 subject to Eq. 5.

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ONE-DIMENSIONAL TEST CASE: AIR-MUSCLE INTERFACE

To test the applicability of the differential-equation-based $(FD)^2TD$ method for dispersive media, we applied it to a one-dimensional problem for the air-2/3 muscle interface. Since the average permittivity of the human body is close to that for 2/3 muscle, we decided to take this permittivity instead of that for the muscle.

A five-relaxation Debye equation given in the following has previously been used to fit to the experimental data for the muscle [6]

$$\varepsilon^* = \varepsilon_o \left[4.3 + \frac{8 \times 10^5}{1 + j\, f/69} + \frac{81{,}900}{1 + j\, f/(43 \times 10^3)} + \frac{11{,}900}{1 + j\, f/(0.67 \times 10^6)} + \frac{32}{1 + j\, f/(230 \times 10^6)} + \frac{45.8}{1 + j\, f/(20 \times 10^9)} \right] \quad (7)$$

where f is the frequency in Hz. Since a five-relaxation Debye equation such as Eq. 7 would result in a fifth-order differential equation for $\mathbf{D}(t)$ in terms of $\mathbf{E}(t)$ and its derivatives, which would require a larger storage of the various quantities for each of the cells, we have attempted to fit a two-relaxation Debye equation similar to Eq. 4 to the experimental data for the muscle. We have been able to obtain a relatively decent fit to the experimental data for the frequency band 20 MHz to 20 GHz by using the following equation:

$$\varepsilon^*(\omega) = \varepsilon_o \left[19 + \frac{10{,}000}{1 + j\, f\left(0.71 \times 10^{-6}\right)} + \frac{42}{1 + j\, f\left(0.75 \times 10^{-10}\right)} \right] \quad (8)$$

Shown in Figs. 1 (a) and (b) as solid curves are the variations given by Eq. 8 for both the relative permittivity ε_r (real part of $\varepsilon^*/\varepsilon_o$) and the conductivity σ (= ω • imaginary part of ε^*). Also shown for comparison are the average values of the experimental data summarized in reference 6 and the variations given by the five-relaxation Debye Eq. 7. For the frequency band 20 MHz to 20 GHz, the variations given by Eq. 8 are in reasonable agreement with the experimental data for the muscle. A major advantage of using the simpler Eq. 8 is that it is possible to use a second-order differential equation for time-domain representation of $\mathbf{D}(t)$ rather than a fifth-order differential equation that would be needed had we used the full-blown Eq. 7 from reference 6.

We divided the problem space into 1000 cells of which 499 were in air and the remaining 501 were in the 2/3 muscle-equivalent material. The cell size taken for the calculations was 0.0127 cm ($<\lambda_g/10$ for $f \leq 40$ GHz) and $\delta t = \delta/2C = 0.21$ ps. The incident Gaussian pulse had a peak amplitude of 1000 V/m and was of the form $E(t) = 1000 \exp\left[-(t - t_o)^2/T^2\right]$ where t_o = 3 T and T = 71 δt. In Fig. 2, we compare the reflection coefficient for the air-2/3 muscle interface obtained for the various frequencies using the $(FD)^2TD$ method with that obtained using the following analytical equation.

$$|R(\omega)| = \left| \frac{\sqrt{\varepsilon_o} - \sqrt{\varepsilon^*(\omega)}}{\sqrt{\varepsilon_o} + \sqrt{\varepsilon^*(\omega)}} \right| \quad (9)$$

Also shown for comparison are the values that would be obtained for the various frequencies had the conventional FDTD method been used and properties assumed corresponding to a midband frequency of, say, 10 GHz. While the $(FD)^2TD$ method using a single run with, say, a Gaussian pulse, gives excellent agreement with the analytical values at all the frequencies, the conventional FDTD method gives agreement for the frequency band where the dielectric properties are close to the values that are assumed for the calculations. As seen in Fig. 2, for dispersive media such as the biological tissues, severe errors of calculated results occur for both lower- and higher-frequency regions.

THREE-DIMENSIONAL TEST CASE: SPHERE OF 2/3 MUSCLE-EQUIVALENT MATERIAL

While no three-dimensional application of the $(FD)^2TD$ has yet been reported in any of the earlier publications [4, 5], we have used the test case of a 2/3 muscle-equivalent sphere and compared the results of the internal E-field distributions at various frequencies with the analytical Mie series solutions that can be obtained at the corresponding frequencies [7]. For the dimensions of the sphere, we have taken a diameter of 20 cm corresponding to the average dimensions of the human head. We have divided the three-dimensional sphere into cells of size 1 cm and have taken a space of 6 cells from the boundaries of the sphere to the absorbing boundaries on each of the sides in the x-, y-, and z-directions, respectively. The time step δt is taken to be 1/60 ns. Because of the larger cell size of 1 cm ($< \lambda_g/10$ for $f \leq 350$ MHz), a Gaussian with lower frequency components is taken as the incident pulse. The assumed Gaussian is of the form $1000 \exp\left[-(t - t_o)^2/T^2\right]$, where t_o = 210 δt and T = 80 δt. In Fig. 3, we compare the $(FD)^2TD$-calculated variation of E_z along the y-axis at a representative frequency with that obtained from the Mie series solution. Though not shown for lack of space, the agreements of the calculated E-field variations using the $(FD)^2TD$ method with the analytical solutions are excellent for frequencies for which the cell size δ is less than $\lambda_g/10$. Similar agreements have also been obtained for other field components for various locations of the sphere.

CALCULATIONS FOR THE HUMAN MODEL

We have used the differential-equation-based $(FD)^2TD$ method to calculate the RF currents induced in the model of the human body. For these calculations we have used the anatomically based model of the human body described in our earlier publications [1-3], except that uniform 2/3 muscle-equivalent ε^* was assumed for each of the cells internal to the body for this first set of calculations. As in the past, volume-averaged properties were used for all of the cells at the model boundary, since these cells are only partially composed of 2/3 muscle-equivalent material, while the rest of it is air. If the tissue content of a given boundary cell is less than 10% by volume, air was assumed to be the material for such cells. For all of the calculations to date we have used cubical cells of size 2.62 cm. As in reference 3, the modeled space is divided into 38 × 26 × 84 = 82,992 cells, of which 5,628 cells are either totally or partially within the human body. A spacing of 7-9 cells from the model is taken to the absorbing boundaries for the various sides of the

modeled space. For the calculations shown in Figs. 4a and 4b, we have assumed isolated (ungrounded) condition of the human model and a flat-top impulse of radiation of width 5 δt in the time-domain, where $\delta t = 0.04367$ ns is the time step used for the calculations. The frequency spectrum of this impulse is fairly broad with a nearly constant amplitude for frequencies up to 500 MHz, which is in excess of the region of validity of this model with cell size of 2.62 cm.

We have used the calculated vertically directed internal D_z fields to calculate local z-directed current densities for each of the cells using the relationship $J_z = \partial D_z/\partial t$. The vertically directed currents for any of the layers were then obtained by summing the terms due to the individual cells in a given layer as follows

$$I = \delta^2 \sum_i \frac{\partial D_z}{\partial t} \qquad (10)$$

where δ^2 is the cross-sectional area (2.62×2.62 cm^2) for each of the cells in the body.

The induced RF current distributions are calculated for some representative frequencies 40, 150, and 350 MHz, and are shown as solid curves in Figs. 4a and 4b. The results for each of the frequencies are normalized for an incident E field of 1 V/m at the corresponding frequencies. Shown for comparison are the current distributions obtained for the same model using the conventional FDTD procedure. Three individual runs have naturally had to be made for the FDTD method, since different properties for ε^* were involved at the three frequencies. As seen in Figs. 4a and 4b, excellent agreement is obtained for the $(FD)^2TD$ results with those obtained using the conventional FDTD method, with the added advantage of being able to use a single run rather than multiple runs needed for the FDTD method.

CONCLUSIONS

In this paper we have described a differential-equation-based frequency-dependent finite-difference time-domain ($(FD)^2TD$) method, which can be used for general dispersive media for which $\varepsilon^*(\omega)$ and $\mu^*(\omega)$ may be expressible in terms of rational functions. We have illustrated the use of this method by one- and three-dimensional examples of media for which $\varepsilon^*(\omega)$ is given by a multiterm Debye equation, and for an approximate two-thirds muscle-equivalent model of the human body. Using a single run involving either a Gaussian pulse or a flat-top impulse, the $(FD)^2TD$ method allows calculations of coupled EM fields and induced currents at various frequencies by taking the Fourier components of the induced fields.

For coupling to extremely narrow pulses such as may be encountered for ultra wideband radar, one can also store the impulse response of the scatterer and convolve the fields thus obtained with the prescribed shape of the incident pulse. This procedure has recently been used [8] to calculate induced currents in the 1.31-cm cell size model of the human body (45,024 cells representing the human body) for exposure to electromagnetic pulses (EMP) and has the advantage of not requiring repeated large-memory FDTD runs.

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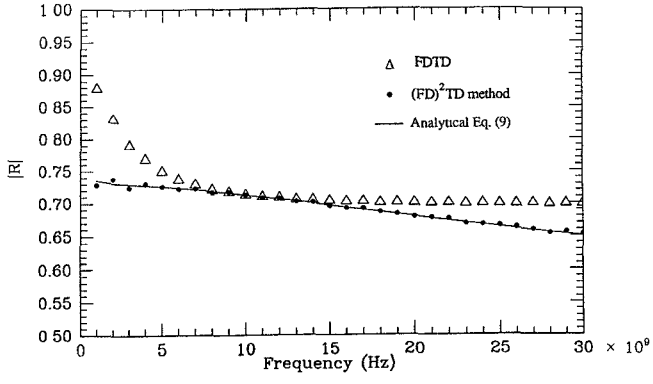

Fig. 2. Comparison of the reflection coefficient for normal incidence at the air-2/3 muscle interface computed using the $(FD)^2TD$ algorithm with broadband pulse irradiation and for the traditional FDTD method where properties at the midband frequency of 10 GHz are taken

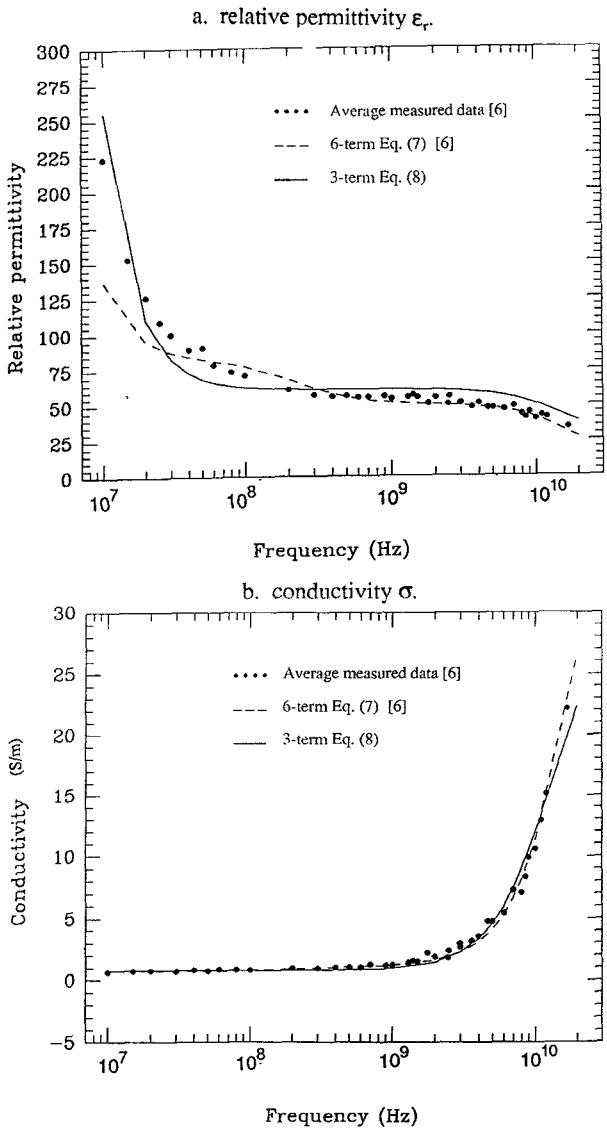

Fig. 1. Frequency variation of the electronic properties of muscle.

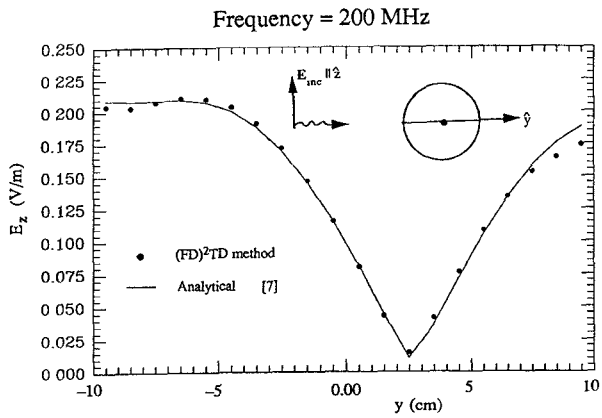

Fig. 3. Comparison of E_z calculated by the $(FD)^2TD$ method with the analytic results using Mie series [7]. Diameter of the 2/3 muscle-equivalent sphere = 20 cm, corresponding to the average dimensions of the human head.

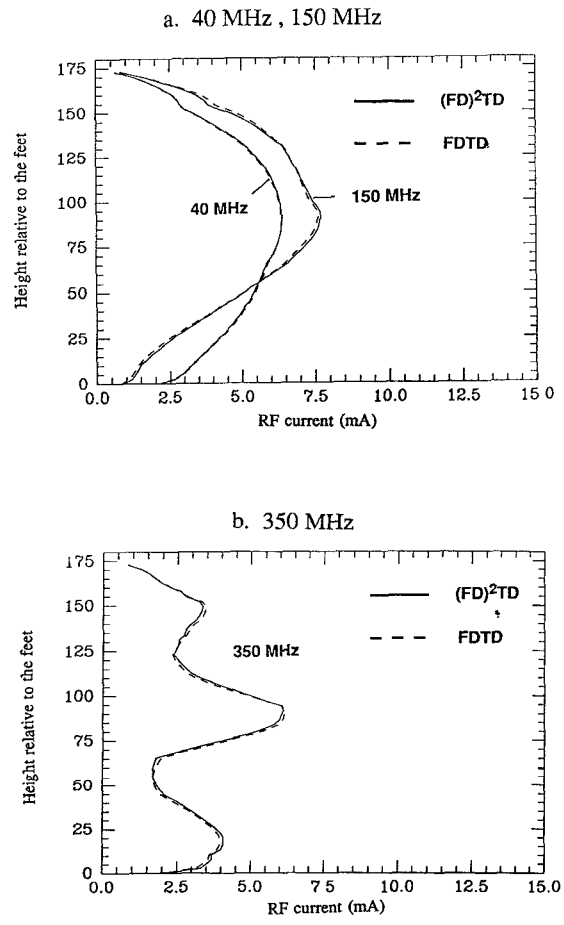

Fig. 4. Induced current distributions for a 2/3 muscle-equivalent model of the human body. Model assumed isolated from ground. E_{inc} = 1 V/m for each of the frequencies.